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The effect of production variables on the strength of brass/Sn–Pb–Sb solder joints: a statistical analysis

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Tomlinson and Cooper's data on solder joint strength have been used to illustrate the additional benefits, in terms of useful results, that can be obtained from the application of stepwise regression techniques to production process data. This technique provides extra quantitative information in three main areas. First, those production variables of importance can be identified and the exact effect of such variables on joint strength determined. Secondly, linear versus non-linear association between such production variables and joint strength can be tested. Finally, the degree and significance of interaction effects can be estimated. Using this stepwise regression technique it was found, contrary to Tomlinson and Cooper's paper, that the relationship between strength and antimony content was, in all cases, non-linear, whilst all other relationships were linear. The significant main effect variables were furnace cooling, cooling time and antimony content with the latter being the most important explanatory variable. However, the effects of these and other interaction explanatory variables were not minor as Tomlinson and Cooper suggest. Important interaction effects were also identified particularly so between gap size and antimony content.

1. Introduction

Statistical techniques can be useful in helping to identify those processes controlling the quality of manufactured items. With reference to data published by Tomlinson and Cooper [1] on solder joint strength, the present work sought to illustrate the nature of the additional information that can be obtained from the application of simple stepwise regression procedures. Tomlinson and Cooper's paper was chosen because it provides a particularly good illustration of the extra useful information that can be obtained through use of this statistical technique.

Tomlinson and Cooper [1] accumulated a moderate data set to analyse the effects of certain production variables on the strength of brass/Sn–Pb–Sb solder joints. Tables and simple line graphs were the methods chosen by the authors to identify those production variables which controlled the strength of the solder joint. (Their tables are reproduced here in the Appendix). Four production variables were examined: solder time (min), percentage content of antimony, joint gap (mm), and cooling method. From such an analysis the authors concluded that "for any solder composition the effect of these production variables is relatively small" and "overall the effect of cooling is small and the effect of joint gap in the range 0.05–0.2 mm has only a minor effect". From the way in which the separate lines in Figs 1 and 3 rotate [1] they conclude that interaction effects between production variables are present in determining joint strength.

Given the chosen methods of analysis, it is not surprising that Tomlinson and Cooper summarized their results in this very qualitative fashion. This present paper illustrates how stepwise regression techniques can be used in conjunction with the above-mentioned graphical methods to obtain the following additional diverse, detailed and precise information.

(a) Those production variables that contribute significantly (in a statistical sense) to joint strength can be found. Further, for those variables found to be significant, their precise effect on strength can be obtained as can their contribution to the variation in joint strength.

(b) The functional relationship existing between joint strength and the production variables can be ascertained. Tests for linear versus non-linear forms can be undertaken. Indeed a graphical plot of strength against solder time (Fig. 1 [1]) is not sufficient information for determining whether or not the relationship is linear.

(c) Those variables that interact and the precise nature of this interaction in determining joint strength can be identified. Graphs alone cannot provide such quantitative detail.

The remainder of the paper illustrates the methods used to obtain this extra information, and the implications that can be drawn from the results.

2. Methodology

The methodology employed here has been referred to in the econometrics literature as Hendrification. This stems from work done by Hendry on dynamic modelling. Hendry [2] suggests that it is always better to start off with a general model and then simplify that model by testing for parameter restrictions. Such tests are termed specification tests, but a simplified model must also pass mis-specification tests, which test the validity of the least squares assumptions about the regression residuals. Specifically, specification tests look for parameter restrictions of the form $\beta_i = 0$, whilst mis-specification tests, analyse the residuals for normality, constant variance and independence.

In applying this approach to data on production variables and joint strength, we come across a number of data constraints. Tomlinson and Cooper look at four production variables: they are soldering time (min), joint gap (mm), percentage of antimony in joints, and cooling method. Four such cooling methods were analysed: water-quenched (WQ), oil-quenched (OQ), air-blown (AB), and furnace-cooled (FC). The cooling times for each were respectively 5 s, 1, 5 min and 6 h. Shear strength (MPa) of the joint was also measured.

To be completely general, all four production variables should be included in a single regression. But as Appendix 1 shows, the way in which the data were collected prevents this. However, three separate regressions can be constructed with the first containing antimony and soldering time, the second antimony and cooling method and the third antimony and gap size. Each can be made general by including powered terms and all interaction variables. Following Mendenhall and Sincich [3] we take as our general specification a "complete factorial model", of which there exist three, given the nature of the data set at hand.

Model 1

$$Y_i = \overbrace{B_0 + B_1T_i + B_2\mathrm{Sb}_i + B_3T_i^2 + B_4\mathrm{Sb}_i^2}^{\text{Main effects}} + \overbrace{B_5T_i\mathrm{Sb}_i + B_6T_i\mathrm{Sb}_i^2 + B_7T_i^2\mathrm{Sb}_i + B_8T_i^2\mathrm{Sb}_i^2}^{\text{Interaction effects}} + \varepsilon_i \quad (1)$$

where Y_i is the ith reading on strength, T_i the ith reading on solder time, Sb_i the percentage antimony content, and $\varepsilon_i \sim N\ [0, \sigma_\varepsilon]$. $P = 1$–8 explanatory variables, and $N = 48$ observations on Y.

Model 2

$$Y_i = \delta_0 + \overbrace{\delta_1\mathrm{Sb}_i + \delta_2\mathrm{Sb}_i^2 + \delta_3\mathrm{OQ} + \delta_4\mathrm{AB} + \delta_5\mathrm{FC}}^{\text{Main effects}}$$

$$+ \overbrace{\delta_6\mathrm{Sb}_i\mathrm{OQ} + \delta_7\mathrm{Sb}_i\mathrm{AB} + \delta_8\mathrm{Sb}_i\mathrm{FC} + \delta_9\mathrm{Sb}_i^2\mathrm{OQ} + \delta_{10}\mathrm{Sb}_i^2\mathrm{AB} + \delta_{11}\mathrm{Sb}_i^2\mathrm{FC}}^{\text{Interaction effects}} + \varepsilon_i' \quad (2a)$$

where OQ = 1 if oil quenched, 0 otherwise. AB = 1 if air blown, 0 otherwise. FC = 1 if furnace cooled, 0 otherwise. $\varepsilon_i' \sim N\ [0, \sigma_\varepsilon']$. $P = 1$–11 variables, and $N = 48$ observations on Y.

This model breaks down into four separate equations, one for each type of cooling.

Water quenched: here OQ = AB = FC = 0, so

$$Y_i = \delta_0 + \delta_1\mathrm{Sb}_i + \delta_2\mathrm{Sb}_i^2 + \varepsilon_i' \quad (2b)$$

Oil quenched: here OQ = 1, AB = FC = 0, so

$$Y_i = [\delta_0 + \delta_3] + [\delta_1 + \delta_6]\mathrm{Sb}_i + [\delta_2 + \delta_9]\mathrm{Sb}_i^2 + \varepsilon_i' \quad (2c)$$

Air blown: here AB = 1, OQ = FC = 0, so

$$Y_i = [\delta_0 + \delta_4] + [\delta_1 + \delta_7]\mathrm{Sb}_i + [\delta_2 + \delta_{10}]\mathrm{Sb}_i^2 + \varepsilon_i' \quad (2d)$$

Furnace cooled: here FC = 1, AB = OQ = 0, so

$$Y_i = (\delta_0 + \delta_5) + [\delta_1 + \delta_8]\mathrm{Sb}_i + [\delta_2 + \delta_{11}]\mathrm{Sb}_i^2 + \varepsilon_i' \quad (2e)$$

Water quenching is the bench mark in Model 2 so that the other types of cooling either shift the quadratic in Equation 2b up or down (e.g. δ_0 to $(\delta_0 + \delta_3)$), and/or alter the slope (e.g. δ_1 to $(\delta_1 + \delta_6)$) and/or alter the degree of curvature (e.g. δ_2 to $(\delta_2 + \delta_9)$).

Model 3

$$Y_i = \overbrace{\alpha_0 + \alpha_1\mathrm{Sb}_i + \alpha_2\mathrm{Sb}_i^2 + \alpha_3\mathrm{GP}_i + \alpha_4\mathrm{GP}_i^2}^{\text{Main effects}} + \overbrace{\alpha_5\mathrm{Sb}_i\mathrm{GP}_i + \alpha_6\mathrm{Sb}_i\mathrm{GP}_i^2 + \alpha_7\mathrm{Sb}_i^2\mathrm{GP}_i + \alpha_8\mathrm{Sb}_i^2\mathrm{GP}_i^2}^{\text{Interaction effects}} + \varepsilon_i'' \quad (3)$$

where GP_i is the ith value on gap size, $\varepsilon_i'' \sim N\ [0, \sigma_\varepsilon'']$, $P = 1$–8 explanatory variables, with $N = 45$ readings only.

The stepwise procedure now runs as follows. First we estimate the general Equations 1–3, and construct the following F test for the omission of each Jth variable

$$F = \frac{[\mathrm{SSR}[X_1, X_2, X_{J-1}, X_{J+1}, \cdots, X_p] - \mathrm{SSR}[X_1, X_2, \cdots, X_P]]}{[\mathrm{MSR}[X_1, X_2, \cdots, X_P]]} \quad (4)$$

This has an F distribution with 1 and N-P-1 degrees of freedom. For Models 1 and 3, $P = 8$, and for Model 2, $P = 11$. X_1 to X_P are the P explanatory variables.

Thus $\mathrm{SSR}[X_1, X_2, X_{J-1}, X_{J+1}, X_P]$ is the sum of squared residuals obtained from a regression with all P explanatory variables except X_J. $\mathrm{MSR}[X_1, X_2, \cdots,$

$X_P]$ is the mean square error of a regression with all P explanatory variables.

Variables are deleted one step at a time, until no more variables can be deleted. Only variables which have low F values will be deleted, as the removal of such variables does not significantly raise the unexplained variation, SSR. In particular, only variables with F values below the critical value at the 10% level of significance, F_c, are dropped.

Once all tests of specification have been carried out, so that no insignificant variables remain, the following three tests of mis-specification are carried out.

1. Form the regression

$$\varepsilon_i = \rho_1\varepsilon_{(i-1)} + \rho_2\varepsilon_{(i-2)} + \cdots + \rho_6\varepsilon_{(i-6)} + \beta_j \sum_{(j=1)}^{P} X_j + Z_i \quad (5)$$

where $Z_i \sim N\ [0, \sigma_z]$. If there is no serial correlation over this range of order, then the value of R^2, (percentage variation in ε_i explained by its own past values and all significant X_j variables), from this regression should be close to zero. In fact, Breusch [4] has shown that the statistic $K_1 = R^2 \times N$, where N is the number of observations on ε_i, follows a χ^2 distribution with six degrees of freedom, under the null hypothesis of independence, i.e. under the null $\rho_1 = \rho_2 = \cdots = \rho_6 = 0$.

2. Form the regression

$$\varepsilon_i^2 = \lambda_0 + \lambda_1 X_1 + \lambda_2 X_1^2 + \cdots + \lambda_{2P-1} X_P + \lambda_{2P} X_P^2 + W_i \quad (6)$$

$W_i \sim N\ [0, \sigma_w]$. ε_i has constant variance if only λ_0 is significant. That is, the R^2 value from this equation should be close to zero. White [5] has shown that the statistic $K_2 = R^2 \times N$, follows a χ^2 distribution with $2P$ degrees of freedom.

3. Jarque and Bera [6] have shown that the following statistic has a χ^2 distribution with 2 degrees of freedom under the null hypothesis of normally distributed residuals

$$K_3 = \frac{[N-P]}{6}\left[SK^2 + \frac{1}{4}EK^2\right] \quad (7)$$

where SK is a measure of skewness and EK is a measure of kurtosis in the residues.

The correct simplified model is one in which no more variables can be deleted by the above F test, and one which passes the K_1 to K_3 tests of mis-specification. Only if the model passes such mis-specification tests, can we use the standard F tests of significance in the simplified model. Least square estimates of parameter variances will be invalid in the presence of non-normality, auto-correlation and heteroscedasticity.

All-regressions were carried out using the Minitab [7] package.

3. Results and discussion

The main results obtained from applying the above procedure on Tomlinson and Cooper's data are shown in Tables I–III and Figs 1–3. Table I shows the results of the analysis when soldering time and antimony are the two explanatory variables. Step 1 shows

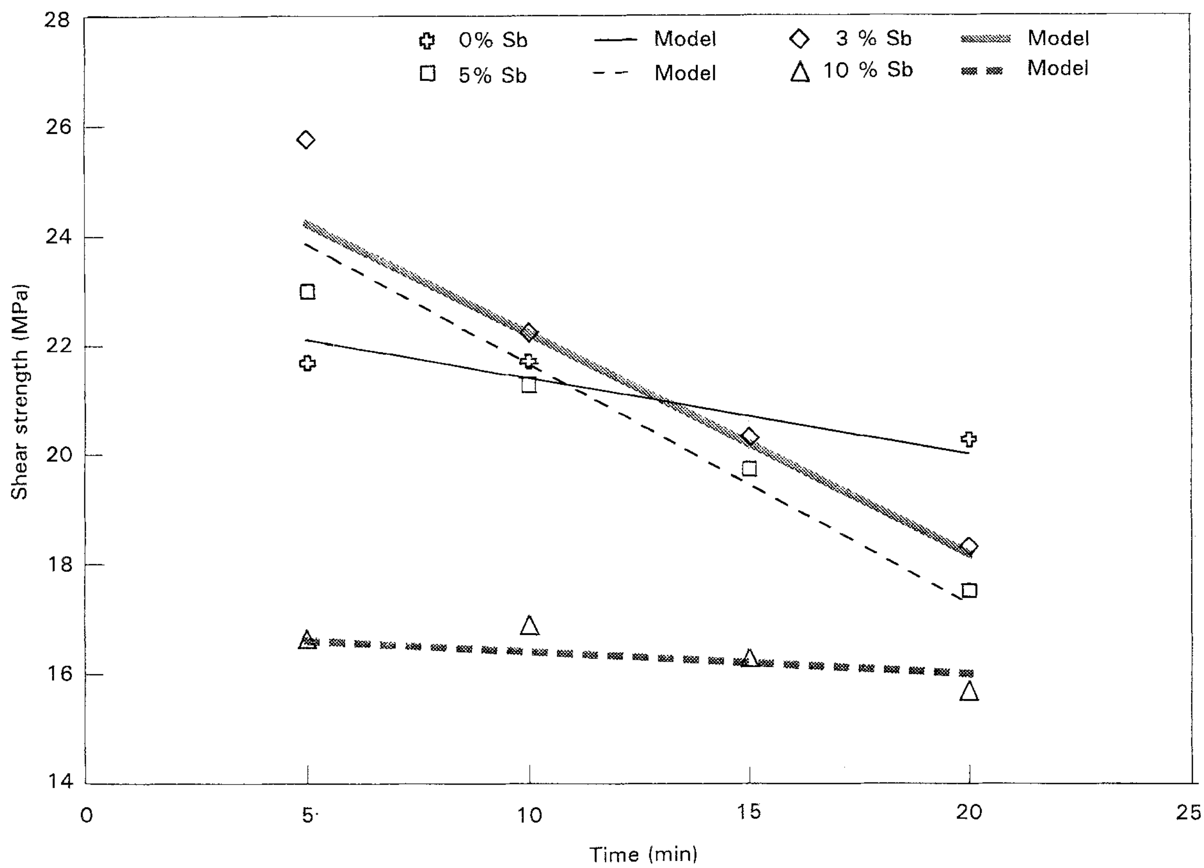

Figure 1 Effect of soldering time at 300 °C on the shear strength of brass/Sn–Pb–Sb joints. Joint gap, D.17 mm; cooled by blown air.

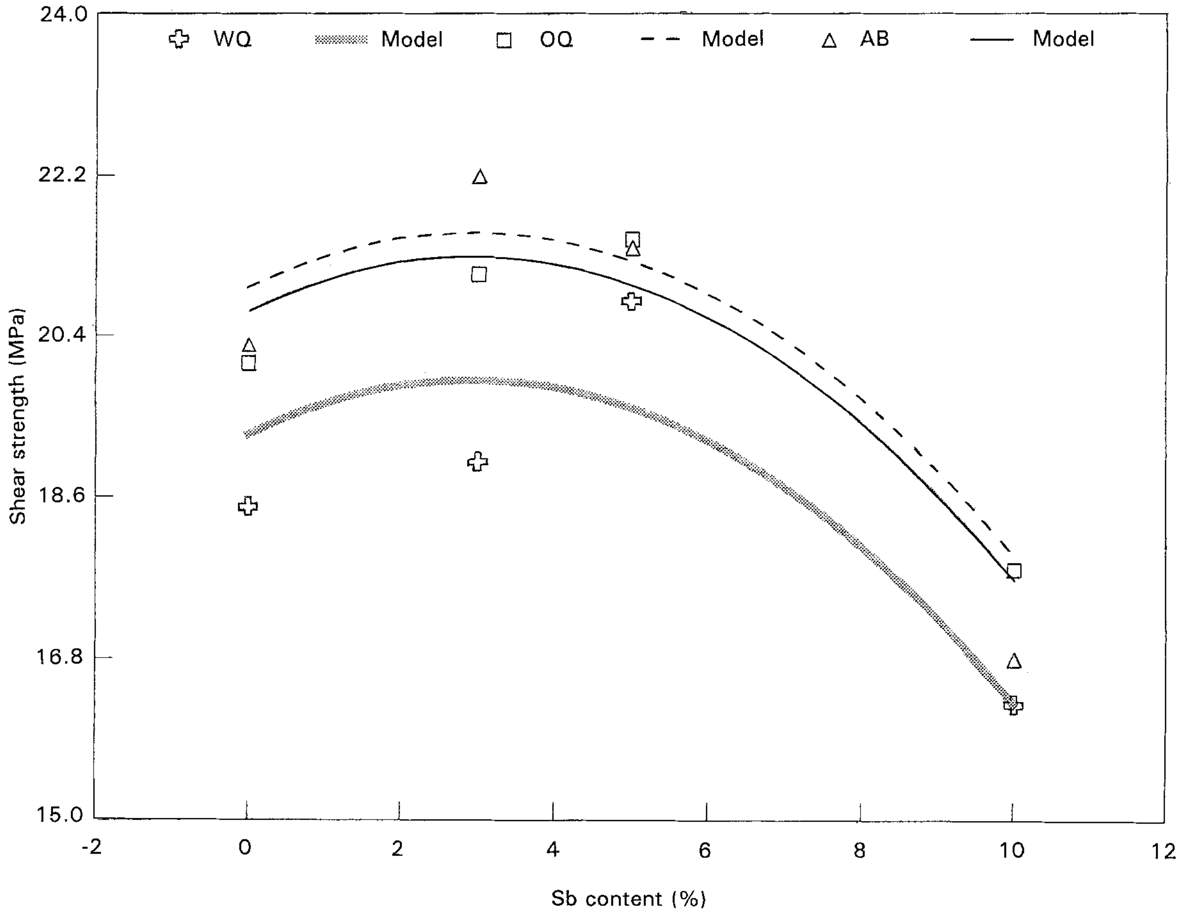

Figure 2 Effect of cooling method on the shear strength of brass/Sn–Pb–Sb joints. Joint gap, 0.17 mm.

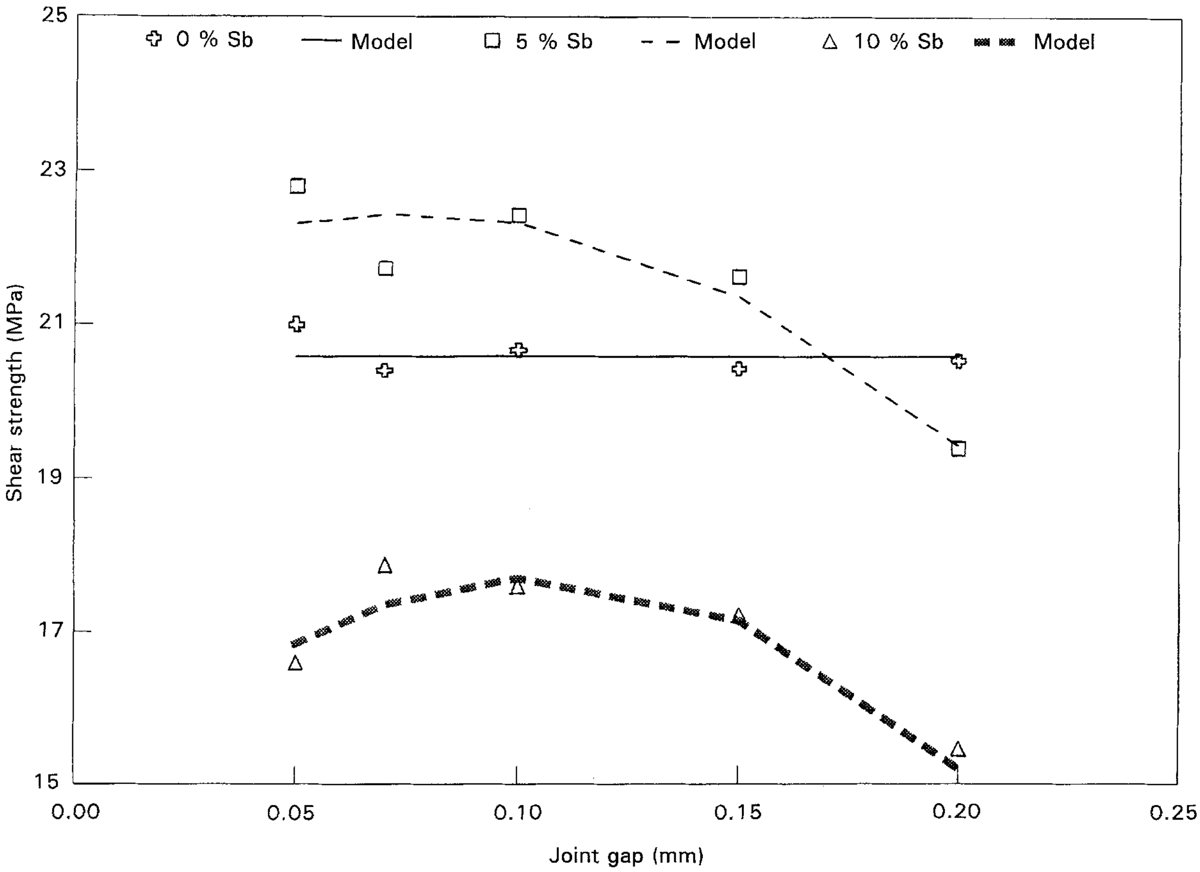

Figure 3 Effect of joint gap on the shear strength of brass/Sn–Pb–Sb joints, soldered at 300 °C for 10 min and cooled by blown air.

TABLE I Stepwise regression for the effects of soldering time and antimony on shear strength of brass joints

Parameter	Step				Mis-specification tests
	1	2	3	4	
Constant	23.2	22.8	22.8	22.8	
F	–	–	–	–	$K_1 = 6.7$
Time, T	− 0.21	− 0.14	− 0.14	− 0.14	$K_2 = 7.7$
F	0.35	5.02	5.11	5.20	$K_3 = 0.7$
Sb	2.1	2.2	1.9	1.9	
F	5.1	10.50	22.28	22.66	
Time², T^2	0.003	–	–	–	
F	0.04	–	–	–	
Sb²	− 0.28	− 0.29	− 0.26	− 0.25	
F	10.8	16.24	39.69	45.02	
Time × Sb	− 0.17	− 0.19	− 0.13	− 0.13	
F	0.96	2.79	18.66	18.92	
Time × Sb²	0.021	0.023	0.016	0.014	
F	1.7	3.13	12.82	25.10	
Time² × Sb	0.002	0.003	–	–	
F	0.06	0.35	–	–	
Time² × Sb²	− 0.0003	− 0.0004	− 0.00008	–	
F	0.21	0.52	− 0.35	–	
R^2 (%)	85.05	85.03	84.90	84.78	
σ_ε	1.24	1.22	1.21	1.20	
F_c (0.1)	2.84	2.844	2.849	2.853	

TABLE II Stepwise regression for the effects of antimony and cooling method on shear strength of brass joints

Parameter	Step								Mis-specification tests
	1	2	3	4	5	6	7	8	
Constant	18.2	18.2	18.7	19.1	19.6	19.1	19.2	19.3	
F	–	–	–	–	–	–	–	–	$K_1 = 1.1$
Sb	0.90	0.88	0.88	0.73	0.57	0.57	0.44	0.42	$K_2 = 4.4$
F	6.05	12.04	11.76	9.49	7.02	7.02	5.24	4.88	$K_3 = 0.1$
OQ	3.60	3.58	3.26	2.73	2.81	2.75	2.07	1.65	
F	11.22	13.18	11.29	9.06	9.36	9.0	7.45	11.36	
AB	2.21	2.17	1.53	1.31	1.55	1.38	1.38	1.38	
F	4.2	9.24	7.78	6.05	9.0	7.95	7.84	7.90	
FC	1.72	1.70	1.38	–	–	–	–	–	
F	2.56	2.96	2.02	–	–	–	–	–	
Sb²	− 0.108	− 0.107	− 0.116	− 0.11	− 0.089	− 0.085	− 0.07	− 0.07	
F	10.18	18.92	23.72	21.16	19.01	18.06	16.97	17.22	
Sb × OQ	− 0.96	− 0.94	− 0.94	− 0.79	− 0.63	− 0.63	− 0.09	–	
F	3.42	4.54	4.45	3.24	2.13	2.13	0.52	–	
Sb × AB	− 0.03	–	–	–	–	–	–	–	
F	0.003	–	–	–	–	–	–	–	
Sb × FC	− 0.94	− 0.93	− 0.93	− 0.48	–	–	–	–	
F	3.31	4.41	4.33	2.25	–	–	–	–	
Sb² × OQ	0.075	0.074	0.083	0.072	0.056	0.052	–	–	
F	2.46	3.17	4.04	3.10	1.90	1.69	–	–	
Sb² × AB	− 0.016	− 0.019	–	–	–	–	–	–	
F	0.12	1.88	–	–	–	–	–	–	
Sb² × FC	0.085	0.083	0.093	0.06	0.01	–	–	–	
F	3.13	4.08	5.06	2.99	1.06	–	–	–	
R^2(%)	69.36	69.36	67.8	66.08	64.13	63.19	61.66	61.20	
σ_ε	1.35	1.33	1.34	1.36	1.38	1.38	1.39	1.39	
F_c (0.1)	2.858	2.862	2.867	2.871	2.876	2.88	2.884	2.889	

the general regression given by Equation 1. The variable with the smallest F value below the critical value of $F_c = 2.84$, is the square of time, T^2. This variable is then dropped for the second step regression. In all, four steps are required to remove all insignificant variables at the 10% level. At the 10% level of significance the model passes all three tests of mis-specification. The simplified model, in which all variables are significant at the 10% level of significance is therefore

$$Y_i = 22.8 - 0.14\,T_i + 1.9\,\mathrm{Sb}_i - 0.25\,\mathrm{Sb}_i^2 - 0.13\,T_i\mathrm{Sb}_i + 0.014\,T_i\mathrm{Sb}_i^2 + \varepsilon_i \qquad (8)$$

From this we can draw the following conclusions.

(a) Joint strength is a linear function of soldering

TABLE III Stepwise regression for the effects of antimony and joint gap on the shear strength of brass joints

Parameter	Step 1	Step 2	Step 3	Step 4	Mis-specification tests
Constant	21.4	21.3	20.8	20.6	
F	–	–	–	–	$K_1 = 9.9$
Sb	0.71	0.78	0.84	0.89	$K_2 = 6.1$
F	0.38	3.57	5.62	8.24	$K_3 = 0.3$
Gap, GP	– 13.1	– 11.9	– 2.0	–	
F	0.10	0.10	0.09	–	
Sb^2	– 0.139	– 0.146	– 0.146	– 0.149	
F	1.61	29.2	29.8	33.4	
Gap^2, GP^2	44	40	–	–	
F	0.07	0.07	–	–	
Sb × Gap	8.8	7.3	6.1	5.9	
F	0.17	1.59	2.72	2.89	
Sb × Gap^2	– 59	– 53	– 48	– 50	
F	0.49	4.19	6.97	8.07	
Gap × Sb^2	– 0.1	–	–	–	
F	0.005	–	–	–	
Sb^2 × Gap^2	2.7	2.1	2.1	2.2	
F	0.11	2.69	2.72	3.84	
$R^2(\%)$	74.06	74.06	74.01	73.94	
σ''_ε	1.42	1.40	1.39	1.37	
F_c (0.1)	2.858	2.862	2.867	2.871	

TABLE IV Explained variations for Model 1, simplified

T_i	24.92%
Sb_i	40.03%
Sb_i^2	9.89%
$T_i Sb_i$	0.84%
$T_i Sb_i^2$	9.10%
All	84.78%

time but a non-linear function of antimony content. This is true at the 10% level of significance.

(b) The variables in Equation 8 jointly explain 84.78% of the total variation in joint strength. 15.2% of the variation remains unexplained. Table IV gives a detailed breakdown of this explained variation. Thus antimony is by far the more important of the explanatory variables, followed by solder time, and Sb^2. That is, the main effects part of the model dominates.

Equation 8 suggests that a one percentage point increase in antimony will induce a 1.65 point MPa increase ($1.9 - 0.25 = 1.65$), all other things being equal. In turn, an increase in soldering time of 1 min induces a 0.14 point MPa decrease, all other things equal. These figures are on the average estimates.

(c) From Table IV, significant interactions take place between time and antimony so as to account for 9.94% of the total variation in joint strength. All percentage figures in Table IV are significant at the 10% level of significance. Thus if solder time goes up by 1 min and antimony by one percentage point, there will be a fall in MPa of 0.116 units above that predicted by the main effects shown in (b) above. This is true on average. This explains why, in Fig. 1 below, the four linear lines rotate as the antimony content is varied. The four linear lines simply plot Equation 8 when Sb = 0%, 3%, 5% and 10%, respectively, and $\varepsilon_i = 0$.

Table II shows the results of the analysis when antimony and cooling method are the two explanatory variables. Step 1 shows the general model given by Equation 2a. In this regression the parameter with the smallest F value below the critical value, $F_c = 2.858$, at the 10% level is Sb × AB. This variable is then dropped for the second step regression. In all, eight steps are required to remove all insignificant variables at the 10% level of significance. Again at the 10% level the simplified model passes all three mis-specification tests.

The simplified model, in which all variables are significant at the 10% level is, therefore

$$Y_i = 19.3 + 0.42\,Sb_i + 1.65\,OQ + 1.38\,AB - 0.07\,Sb_i^2 + \varepsilon_i' \quad (9a)$$

From this we can draw the following conclusions.

(a) Joint strength is a non-linear function (quadratic) of antimony content. This is true at the 10% level of significance.

(b) From Equation 9a we have

$$Y_i = 19.3 + 0.42\,Sb_i - 0.07\,Sb_i^2 + \varepsilon_i' \quad (9b)$$

when water quenching is used.

$$Y_i = 20.95 + 0.42\,Sb_i - 0.07\,Sb_i^2 + \varepsilon_i' \quad (9c)$$

when oil quenching is used.

$$Y_i = 20.68 + 0.42\,Sb_i - 0.07\,Sb_i^2 + \varepsilon_i' \quad (9d)$$

when air-blown cooling is used. At the 10% level of significance Equations 9b–d are significantly different. Excluding furnace cooling, changes in cooling method shift the quadratic relationship between strength and antimony content upwards in a parallel fashion. However, no matter what type of cooling is used, every one percentage point rise in antimony will add to MPa

TABLE V Explained variation for Model 2, simplified

Sb_i	32.52%
Sb_i^2	15.51%
OQ	6.02%
AB	7.15%
All	61.2%

some 0.28 units. That is

$$\frac{dY}{dSb} = 0.42 - 0.14Sb \quad (10)$$

This holds on average.

The variables in Equation 9a explain 61.2% of the variation in joint strength. Some 38.8% of the variation remains unexplained. Table V shows a more detailed breakdown of this explained variation. Sb_i and its square are by far the most important explanatory variables. However, the methods of cooling are capable of explaining some 13.17% of the variation in joint strength. This is significant at the 10% level.

(c) Equations 9b–d highlight the fact that no significant interactions take place. The cooling method simply alters the constant term in Equation 9a. That is, changes in the cooling method shift the quadratic in strength–antimony space in a parallel manner. This is shown in Fig. 2 in which Equations 9a–d are plotted for $\varepsilon_i' = 0$.

Table III shows the results of the analysis when antimony and gap size are the explanatory variables. Step 1 shows the general model given by Equation 3. The variable with the smallest F value below the critical value of $F_c = 2.858$, is $GP \times Sb^2$. This variable is then dropped for the second step regression. In all, four steps are needed to remove all insignificant variables at the 10% level. At this 10% level of significance the simplified model passes all three mis-specification tests. The simplified model in which all variables are significant at the 10% level is given by

$$Y_i = 20.6 + 0.89\,Sb_i - 0.149\,Sb_i^2 + 5.9\,Sb_i \times GP_i - 50\,Sb_i \times GP_i^2 + 2.2\,Sb_i^2 GP_i^2 + \varepsilon_i'' \quad (11)$$

From this we can draw the following conclusions.

(a) Joint strength is a non-linear function (quadratic) of antimony content, but a linear function of gap size. Thus the curves shown in Fig. 3 of Tomlinson and Cooper's article are not curves at all, at the 10% level of significance. Evidence for divergence from a straight line is not present in the data.

(b) The variables in Equation 11 jointly explain 73.94% of the total variation in joint strength. 26.06% of the variation in joint strength remains unexplained. Table VI contains a further breakdown of this explained variation. Again Sb and Sb^2 are by far the most important explanatory variables. The main effects part of the model dominates. Gap size as a variable on its own does not significantly affect joint strength, although it does have an interaction effect with antimony. Ignoring such interaction, an increase of one percentage point in antimony adds 0.592 units

TABLE VI Explained variation for Model 3, simplified

Sb_i	35.55%
Sb_i^2	28.24%
$Sb_i \times GP_i$	4.66%
$Sb_i \times GP_i^2$	2.93%
$Sb_i^2 \times GP_i^2$	2.56%
All	73.94%

to the MPa measure. That is

$$\frac{dY}{dSb} = 0.89 - 0.298Sb \quad (12)$$

These are, on average, estimates.

(c) Table VI shows that significant interaction effects take place between Sb and GP, Sb and GP^2 and Sb^2 and GP^2 on strength. All such interactions help explain 10.15% of the variation in joint strength. Through such interactions a 0.1 mm rise in gap size with a one percentage point rise in antimony will add 0.112 MPa to joint strength on average. This helps explain why in Fig. 3 the three linear lines rotate as antimony varies. These three linear lines simply plot Equation 11 when Sb = 0%, 5% and 10%, respectively, with $\varepsilon_i'' = 0$.

4. Conclusion

The results illustrate that the percentage content of antimony is the most important production variable determining joint strength. However, the effects of other variables are far from minor. In Model 1, solder time helps explain nearly a quarter of the variation in joint strength, whilst in Model 2, some 13% of joint

TABLE VII Effect of soldering time at 300 °C on the shear strength of brass/Sn–Pb–Sb joints. Joint gap 0.17 mm; cooled by blown air

Sb (%)	Time (min)	Shear strength (MPa)			
		1	2	3	Average
0	5	22.9	21.5	20.6	21.6
0	10	22.8	21.7	20.6	21.7
0	15	19.6	19.4	21.7	20.3
0	20	19.4	21.3	20.1	20.3
0	60	16.3	–	–	16.3
3	5	26.7	24.4	26.2	25.7
3	10	22.1	22.9	21.7	22.2
3	15	20.1	18.7	22.1	20.3
3	20	19.1	17.1	18.7	18.3
3	60	16.1	–	–	16.1
5	5	24.4	21.3	23.3	23.0
5	10	22.3	22.1	19.4	21.3
5	15	19.6	21.3	18.3	19.8
5	20	18.6	15.8	18.1	17.5
5	60	15.9	–	–	15.9
10	5	15.2	17.6	17.1	16.7
10	10	17.1	17.3	16.3	16.9
10	15	16.3	17.4	15.2	16.3
10	20	16.4	15.2	15.5	15.7
10	60	15.7	–	–	15.7

strength variation is explained by cooling methods. In Model 3 interaction effects were shown to be very important. Thus, whilst gap size on its own is irrelevant, its interaction with antimony helps explain 10% of the variation in joint strength. Finally, joint strength was shown to be a linear function of solder time and gap size, not non-linear as Tomlinson and Cooper suggest, and a non-linear function of antimony content.

These simple statistical procedures have helped quantify and identify the important and not so important production variables and, in some cases, even modified some of Tomlinson and Cooper's conclusions. It is clear that a whole wealth of extra information can be gleamed from the authors' data with the use of this simple regression procedure.

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TABLE VIII Effect of cooling rate after soldering time at 300 °C for 10 min on the shear strength of brass/Sn–Pb–Sb joints. Joint gap = 0.17 mm; WQ = water quenched, OQ = oil quenched, AB = air blown, FC = furnace cooled

Sb (%)	Cooling method	Shear strength (MPa)			
		1	2	3	Average
0	WQ	17.6	19.5	18.3	18.5
0	OQ	20.0	24.3	21.9	20.1
0	AB	18.3	19.8	22.9	20.3
0	FC	19.4	19.8	20.3	19.8
3	WQ	18.6	19.5	19.0	19.0
3	OQ	20.0	20.9	20.4	21.1
3	AB	21.7	22.9	22.1	22.2
3	FC	19.0	20.9	19.9	19.7
5	WQ	22.3	19.5	20.5	20.8
5	OQ	20.9	22.9	20.6	21.5
5	AB	22.9	19.7	21.6	21.4
5	FC	19.6	16.4	20.5	18.8
10	WQ	15.2	17.1	16.6	16.3
10	OQ	16.4	19.0	18.1	17.8
10	AB	15.8	17.3	17.1	16.8
10	FC	16.4	17.6	17.6	17.2

TABLE IX Effect of joint gap on the shear strength of brass/Sn–Pb–Pb–Sb joints soldered at 300 °C for 10 min and cooled by blown air

Sb (%)	Gap (mm)	Shear strength (MPa)			
		1	2	3	Average
0	0.05	19.6	23.0	20.4	21.0
0	0.07	18.5	20.8	21.9	20.4
0	0.10	19.0	21.9	21.1	20.7
0	0.15	22.3	19.0	20.0	20.4
0	0.20	17.8	22.9	20.9	20.5
5	0.05	20.1	23.5	24.8	22.8
5	0.07	22.9	22.5	19.8	21.7
5	0.10	22.5	22.9	21.9	23.4
5	0.15	20.6	20.5	23.8	21.7
5	0.20	19.0	20.5	18.7	19.4
10	0.05	16.0	16.3	17.5	16.6
10	0.07	18.7	17.1	17.8	17.9
10	0.10	19.0	17.1	16.7	17.6
10	0.15	17.6	17.0	17.1	17.3
10	0.20	16.2	15.2	15.0	15.5

Appendix
Tables VII–IX give the original data published by Tomlinson and Cooper. As can be seen, three MPa observations are made against two explanatory variables in each table. Thus only two such variables can be included in any one regression equation

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